



Factoring

Factoring the Difference of Two Squares

$$a^2 - 36 = (a + 6)(a - 6)$$
$$3x^2 - 48 = 3(x^2 - 16) = 3(x + 4)(x - 4)$$

Factor, write prime if prime.

1. $x^2 - 1$
2. $x^2 - 9$
3. $x^2 + 4$
4. $x^2 - 25$
5. $9y^2 - 16$
6. $4x^2 - 25$
7. $9x^2 - 1$
8. $a^2 - x^2$
9. $25 - m^2$
10. $x^2 - 16y^2$
11. $25m^2 - n^2$
12. $-x^2 + 16$
13. $36m^2 - 121$
14. $2x^2 - 8$
15. $25 + 4x^2$
16. $4a^2 - 81b^2$
17. $12x^2 - 75$
18. $a^2b - b^3$
19. $-98 + 2x^2$
20. $5x^2 - 45y^2$
21. $9x^4 - 4$
22. $16x^4 - y^2$



Factoring Perfect Square Trinomials

$$x^2 - 14x + 49 = (x - 7)^2$$

Factor, write prime if prime.

1. $x^2 + 8x + 16$

11. $25a^2 + 60a + 36$

2. $x^2 - 16x + 64$

12. $16 + 40x + 25x^2$

3. $y^2 + 12y + 36$

13. $16x^2 + 24x + 9$

4. $a^2 - 10a + 25$

14. $49x^2 - 14x + 1$

5. $16y^2 + 8y + 1$

15. $9y^2 - 30y + 25$

6. $9x^2 - 6x + 1$

16. $n^2 + 2n + 4$

7. $25x^2 + 10x + 1$

17. $b^2 + 2b + 1$

8. $n^2 - 14n + 49$

18. $36x^2 + 84x + 49$

9. $81x^2 - 90x + 25$

19. $81 - 18x + x^2$

10. $4y^2 - 20y + 25$

20. $4 - 12y + 9y^2$



Factoring

Extra: Factoring by Grouping

$$\begin{aligned}6ax - 2b - 3a + 4bx &= 6ax - 3a + 4bx - 2b \\ &= 3a(2x - 1) + 2b(2x - 1) \\ &= (2x - 1)(3a + 2b)\end{aligned}$$

1. $x^2 + 2x + xy + 2y$

8. $n^2 + 2n + 3mn + 6m$

2. $3a^2 - 2b - 6a + ab$

9. $2ax^2 + bx^2 - 2ay^2 - by^2$

3. $t^3 - t^2 + \underbrace{t - 1}$

Hint: $t - 1 = 1(t - 1)$

10. $yz^2 - y^3 + z^3 - y^2z$

4. $10 + 2t - 5s - st$

11. $y^3 - y^2 - 4y + 4$

5. $\frac{2}{3}bc - \frac{14}{3}b + c - 7$

12. $x^2a + x^2b - 16a - 16b$

6. $4u^2 + v + 2uv + 2u$

13. $x^3 + x^2 - x - 1$

7. $ad + 3a - d^2 - 3d$

14. $a^3 - a^2 - 8a + 8$



Factoring: Putting It All Together

$$5x^2 + 20x - 60 = 5(x^2 + 4x - 12) = 5(x + 6)(x - 2)$$

Factor completely, write prime if prime.

1. $2x^2 - 8$

9. $4x^2 + 16x + 16$

2. $2x^2 + 8x + 6$

10. $18x + 12x^2 + 2x^3$

3. $3n^2 + 9n - 30$

11. $2x - 2xy^2$

4. $6x^2 - 26x - 20$

12. $3t^3 - 27t$

5. $2x^2 + 12x - 80$

13. $24a^2 - 30a + 9$

6. $5t^2 + 15t + 10$

14. $10x^2 + 15x - 10$

7. $8n^2 - 18$

15. $3x^2 - 42x + 147$

8. $14x^2 + 7x - 21$

16. $4x^4 - 4x^2$



Solving Equations Using Factoring

1. Rewrite equation in standard form (one member equals 0).
2. Factor completely.
3. Set each factor equal to 0; then solve.
4. Check results in original equation.

$$x^2 - 7x + 12 = 0$$

$$(x - 4)(x - 3) = 0$$

$$x - 4 = 0 \text{ or } x - 3 = 0$$

$$x = 4 \quad x = 3$$

$$x = 3, 4$$

$$v^3 = 10v - 3v^2$$

$$v^3 + 3v^2 - 10v = 0$$

$$v(v^2 + 3v - 10) = 0$$

$$v(v + 5)(v - 2) = 0$$

$$v = 0 \text{ or } v + 5 = 0 \text{ or } v - 2 = 0$$

$$v = -5 \quad v = 2$$

$$v = -5, 0, 2$$

1. $x^2 - 5x - 6 = 0$

9. $23p = 5p^2 + 24$

2. $v^3 - 4v = 0$

10. $x^2 - 3x - 10 = 0$

3. $n^2 - 16n = 0$

11. $y^2 = 49$

4. $x^2 + 9 = 10x$

12. $y^2 = -7y - 10$

5. $6x^2 = 16x - 8$

13. $x^2 = 8x$

6. $s^2 = 56s - s^3$

14. $3x^2 - 2 = x^2 + 6$

7. $3y^2 + 2y - 1 = 0$

15. $4y^2 = -4y - 1$

8. $u^3 = 14u^2 + 32u$

16. $5x^2 - 2x - 3 = 0$

Factoring Worksheet

Remember these steps for factoring polynomials.

- 1) Is there a common factor? If so factor it out.
- 2) How many terms in the polynomial?
Two terms – check to see if it is the difference of squares, the difference of cubes, or the sum of cubes
Three terms – Check to see if it is a perfect square trinomial. If not use the a-c factoring method or the hunt and guess method.
Four or more terms – Try to factor by grouping
- 3) Once you have factored, can any of the factors be factored further? If so continue factoring

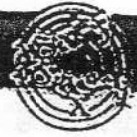
Use the above techniques to factor the polynomials completely.

- | | | |
|----------------------------|-----------------------------|------------------------|
| 1. $a^2 - 4a - 12$ | 13. $8a^5 - 8a^4 - 48a^3$ | |
| 2. $a^2 + 17a + 72$ | 14. $8k^2 - 10k - 3$ | |
| 3. $6y^2 - 6y - 12$ | 15. $x^4 - 16$ | |
| 4. $7y^6 + 14y^5 - 168y^4$ | 16. $50z^2 - 100$ | |
| 5. $6a + 12b + 18c$ | 17. $x^2 - 4x - 5x + 20$ | |
| 6. $m^2 - 3mn - 4n^2$ | 18. $ab^2 + ba^2 - 5a - 5b$ | |
| 7. $p^2 - 17p + 66$ | 19. $6n^2 - 19n + 10$ | |
| 8. $z^2 - 6z + 7z - 42$ | 20. $9y^2 + 12y - 5$ | |
| 9. $10z^2 - 7z - 6$ | 21. $16x + 20$ | |
| 10. $2m^2 - 10m - 48$ | 22. $4z^3 - 36$ | |
| 11. $t^3 + 27$ | 23. $6y^2 - 5y - 4$ | |
| 12. $15y + 5$ | 24. $m^2 - 81$ | 25. $12z^2 + 62z + 10$ |

Solutions

1. $(a - 6)(a + 2)$
2. $(a + 8)(a + 9)$
3. $6(y - 2)(y + 1)$
4. $7y^4(y + 6)(y - 4)$
5. $6(a + 2b + 3c)$
6. $(m - 4n)(m + n)$
7. $(p - 11)(p - 6)$
8. $(z + 7)(z - 6)$
9. $(5z - 6)(2z + 1)$
10. $2(m - 8)(m + 3)$

11. $(t + 3)(t^2 - 3t + 9)$
12. $5(3y + 1)$
13. $8a^3(a - 3)(a + 2)$
14. $(4k + 1)(2k - 3)$
15. $(x - 2)(x + 2)(x^2 + 4)$
16. $50(z^2 - 2)$
17. $(z - 5)(x - 4)$
18. $(a + b)(ab - 5)$
19. $(3n - 2)(2n - 5)$
20. $(3y - 1)(3y + 5)$
21. $4(4x + 5)$
22. $4(z^3 - 9)$
23. $(3y - 4)(2y + 1)$
24. $(m + 9)(m - 9)$
25. $2(6z + 1)(z + 5)$



Factoring Perfect Square Trinomials

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Factor, write prime if prime.

- | | |
|-------------------------------------|--------------------------------------|
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$(x+4)^2$ | 11. $25a^2 + 60a + 36$
$(5a+6)^2$ |
| 2. $x^2 - 16x + 64$
$(x-8)^2$ | 12. $16 + 40x + 25x^2$
$(4+5x)^2$ |
| 3. $y^2 + 12y + 36$
$(y+6)^2$ | 13. $16x^2 + 24x + 9$
$(4x+3)^2$ |
| 4. $a^2 - 10a + 25$
$(a-5)^2$ | 14. $49x^2 - 14x + 1$
$(7x-1)^2$ |
| 5. $16y^2 + 8y + 1$
$(4y+1)^2$ | 15. $9y^2 - 30y + 25$
$(3y-5)^2$ |
| 6. $9x^2 - 6x + 1$
$(3x-1)^2$ | 16. $n^2 + 2n + 4$
prime |
| 7. $25x^2 + 10x + 1$
$(5x+1)^2$ | 17. $b^2 + 2b + 1$
$(b+1)^2$ |
| 8. $n^2 - 14n + 49$
$(n-7)^2$ | 18. $36x^2 + 84x + 49$
$(6x+7)^2$ |
| 9. $81x^2 - 90x + 25$
$(9x-5)^2$ | 19. $81 - 18x + x^2$
$(x-9)^2$ |
| 10. $4y^2 - 20y + 25$
$(2y-5)^2$ | 20. $4 - 12y + 9y^2$
$(3y-2)^2$ |



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Extra: Factoring by Grouping

$$\begin{aligned}
 6ax - 2b - 3a + 4bx &= 6ax - 3a + 4bx - 2b \\
 &= 3a(2x - 1) + 2b(2x - 1) \\
 &= (2x - 1)(3a + 2b)
 \end{aligned}$$

1. $x^2 + 2x + xy + 2y$

$(x+2)(x+y)$

8. $n^2 + 2n + 3mn + 6m$

$(n+3m)(n+2)$

2. $3a^2 - 2b - 6a + ab$

$(a+2)(3a+b)$

9. $2ax^2 + bx^2 - 2ay^2 - by^2$

$(x-y)(x+y)(2a+b)$

3. $t^3 - t^2 + t - 1$

Hint: $t - 1 = 1(t - 1)$

$(t-1)(t^2+1)$

10. $yz^2 - y^3 + z^3 - y^2z$

$(z-y)(z+y)^2$

4. $10 + 2t - 5s - st$

$(2-s)(t+5)$

11. $y^3 - y^2 - 4y + 4$

$(y+2)(y-2)(y-1)$

5. $\frac{2}{3}bc - \frac{14}{3}b + c - 7$

$(c-7)(\frac{2}{3}b+1)$

12. $x^2a + x^2b - 16a - 16b$

$(x+4)(x-4)(a+b)$

5. $4u^2 + v + 2uv + 2u$

$(2u+1)(2u+v)$

13. $x^3 + x^2 - x - 1$

$(x+1)^2(x-1)$

7. $ad + 3a - d^2 - 3d$

$(a-d)(d+3)$

14. $a^3 - a^2 - 8a + 8$

$(a-1)(a^2-8)$

Factoring: Putting It All Together

$$5x^2 + 20x - 60 = 5(x^2 + 4x - 12) = 5(x+6)(x-2)$$

Factor completely, write prime if prime.

1. $2x^2 - 8$

$2(x+2)(x-2)$

9. $4x^2 + 16x + 16$

$4(x+2)^2$

2. $2x^2 + 8x + 6$

$2(x+3)(x+1)$

10. $18x + 12x^2 + 2x^3$

$2x(x+3)^2$

3. $3n^2 + 9n - 30$

$3(n+5)(n-2)$

11. $2x - 2xy^2$

$2x(1+y)(1-y)$

4. $6x^2 - 26x - 20$

$2(3x+2)(x-5)$

12. $3t^3 - 27t$

$3t(t+3)(t-3)$

5. $2x^2 + 12x - 80$

$2(x+10)(x-4)$

13. $24a^2 - 30a + 9$

$3(2a-1)(4a-3)$

6. $5t^2 + 15t + 10$

$5(t+1)(t+2)$

14. $10x^2 + 15x - 10$

$5(2x-1)(x+2)$

7. $8n^2 - 18$

$2(2n+3)(2n-3)$

15. $3x^2 - 42x + 147$

$3(x-7)^2$

8. $14x^2 + 7x - 21$

$7(2x+3)(x-1)$

16. $4x^4 - 4x^2$

$(4x^2)(x+1)(x-1)$



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Factoring the Difference of Two Squares

$$a^2 - 36 = (a + 6)(a - 6)$$
$$3x^2 - 48 = 3(x^2 - 16) = 3(x + 4)(x - 4)$$

Factor, write prime if prime.

- $x^2 - 1$
 $(x+1)(x-1)$
- $x^2 - 9$
 $(x+3)(x-3)$
- $x^2 + 4$
prime
- $x^2 - 25$
 $(x+5)(x-5)$
- $9y^2 - 16$
 $(3y+4)(3y-4)$
- $4x^2 - 25$
 $(2x+5)(2x-5)$
- $9x^2 - 1$
 $(3x+1)(3x-1)$
- $a^2 - x^2$
 $(a+x)(a-x)$
- $25 - m^2$
 $(5+m)(5-m)$
- $x^2 - 16y^2$
 $(x+4y)(x-4y)$
- $25m^2 - n^2$
 $(5m+n)(5m-n)$
- $-x^2 + 16$
 $(4+x)(4-x)$
- $36m^2 - 121$
 $(6m+11)(6m-11)$
- $2x^2 - 8$
 $2(x+2)(x-2)$
- $25 + 4x^2$
prime
- $4a^2 - 81b^2$
 $(2a+9b)(2a-9b)$
- $12x^2 - 75$
 $3(2x+5)(2x-5)$
- $a^2b - b^3$
 $b(a+b)(a-b)$
- $-98 + 2x^2$
 $2(x+7)(x-7)$
- $5x^2 - 45y^2$
 $5(x+3y)(x-3y)$
- $9x^4 - 4$
 $(3x^2+2)(3x^2-2)$
- $16x^4 - y^2$
 $(4x^2+y)(4x^2-y)$